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Differential Calculus (Partial derivatives)

Q.1. If $x^3+y^3-x^3y^2z=0$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $x=y=1$

Soln. We have, $x^3+y^3-x^3y^2z=0$

$$\Rightarrow x^3+y^3=x^3y^2z \Rightarrow z=\frac{x^3+y^3}{x^3y^2}=\frac{1}{y^2}+\frac{y}{x^3}$$

$$\therefore \frac{\partial z}{\partial x}=-3\frac{y}{x^4},\quad \frac{\partial z}{\partial y}=-\frac{2}{y^3}+\frac{1}{x^3}$$

$\therefore$ when $x=y=1$

$$\frac{\partial z}{\partial x}=-3,\quad \frac{\partial z}{\partial y}=-2+1=-1.$$

Q.2. If $u=\begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$

Soln. We have $u=\begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$

$$\Rightarrow u=x^2(y-z)+y^2(z-x)+z^2(x-y) \quad \text{---(I)}$$

Differentiating I partially with respect to x, keeping y and z constants, we get

$$\frac{\partial u}{\partial x}=2x(y-z)+y^2(-1)+z^2\cdot 1=2xy-2zx-y^2+z^2 \quad \text{---(II)}$$

Similarly, $\frac{\partial u}{\partial y}=x^2\cdot 1+2y(z-x)+z^2(-1)=x^2+2yz-2xy-z^2$ ---(III)

and $\frac{\partial u}{\partial z}=x^2(-1)+y^2\cdot 1+2z(x-y)=-x^2+y^2+2zx-2yz$ ---(IV)

Adding II, III and IV, we get $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ proved

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Differential Calculus

Q.3. If $u = \frac{x^2+y^2}{x+y}$, prove that

$$\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left\{1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right\}$$

Soln: Here, $u = \frac{x^2+y^2}{x+y}$ —— (1)

Differentiating (1) partially with respect to x keeping y constants, we get

$$\frac{\partial u}{\partial x} = \frac{2x(x+y) - 1\cdot(x^2+y^2)}{(x+y)^2} = \frac{x^2+2xy-y^2}{(x+y)^2}$$

Similarly, differentiating I partially w.r.t. y, we get

$$\frac{\partial u}{\partial y} = \frac{2y(x+y) - 1\cdot(x^2+y^2)}{(x+y)^2} = \frac{2xy+y^2-x^2}{(x+y)^2}$$

$$\therefore \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = \frac{x^2+2xy-y^2}{(x+y)^2} - \frac{2xy+y^2-x^2}{(x+y)^2}$$

$$= \frac{x^2+\cancel{2xy}-y^2-\cancel{2xy}-y^2+x^2}{(x+y)^2} = \frac{2(x^2-y^2)}{(x+y)^2}$$

$$= \frac{2\cancel{(x+y)}(x-y)}{\cancel{(x+y)^2}}$$

$$= \frac{2(x-y)}{x+y}$$

$$\therefore L.H.S = \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2$$

$$= \frac{4(x-y)^2}{(x+y)^2}$$

Again, $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{x^2+2xy-y^2}{(x+y)^2} + \frac{2xy+y^2-x^2}{(x+y)^2} = \frac{\cancel{x^2}+2xy-\cancel{y^2}+2xy+\cancel{y^2}-\cancel{x^2}}{(x+y)^2}$

$$= \frac{4xy}{(x+y)^2}$$

$$\therefore R.H.S = 4\left\{1 - \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right\}$$

$$= 4\left\{1 - \frac{4xy}{(x+y)^2}\right\} = 4\frac{\{(x+y)^2 - 4xy\}}{(x+y)^2} = 4\frac{(x-y)^2}{(x+y)^2}$$ Ans.